O. P. JINDAL SCHOOL, SAVITRI NAGAR
Half Yearly Examination (2025 – 2026)

Class: XII | MM: 80
Subject: Mathematics | Time: 3 Hrs.

Fifteen Minutes Extra will be for reading the Question Paper.

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

This section comprises of Multiple Choice Questions (MCQ) of 1 mark each

1. The derivative $y = \log(\cos e^x)$ w.r.t. x is:

   (a) $\tan e^x$ (b) $e^x \tan e^x$ (c) $-e^x \tan e^x$ (d) None of these

2. If $x = a(1 - \cos\theta)$, $y = a(\theta - \sin\theta)$, then value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ is :

   (a) $\frac{1}{2}$ (b) -1 (c) 1 (d) None of these

3. If $y = \sin^{-1} x$, then $(1 - x^2)y_2$ *is* equal to

   (a) xy (b) xy_2 (c) xy_1 (d) x^2

4. The value of 'k' for which the function f(x) defined by

   $$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$, is continuous at x = 0 is :

   (a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) None of these

5. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of a.

   (a) 100 (b) 120 (c) 140 (d) 160

6. Find the intervals in which $f(x) = x^2 + 2x - 5$ is strictly increasing.

   (a) x>1 (b) x<-1 (c) x>-1 (d) x>2

7. What is the nature of function $f(x) = x^3 - 3x^2 + 4x$ on R?

   (a) Increasing (b) Decreasing (c) Constant (d) Increasing and Decreasing

8. The rate of change of area of a square is 40 cm^2/s. What will be the rate of change of side if the side is 10 cm.

   (a) 2 cm/s (b) 4 cm/s (c) 8 cm/s (d) 6 cm/s

9. The total cost P(x) in rupees associated with a product is given by $P(x)=0.4x^2+2x-10$. Find the marginal cost if the no. of units produced is 5.

   (a) Rs.3 (b) Rs.4 (c) Rs.5 (d) Rs.6

10. Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) π (d) None of these

11. Evaluate: $\int \cos x \, e^{\sin x} dx$

(a) $e^{\sin x} + C$ (b) $e^{\cos x} + C$ (c) $\sin x e^{\cos x} + C$ (d) $-e^{\sin x} + C$

12. Evaluate: $\int \frac{e^x}{1+e^{2x}} dx$

(a) $\tan^{-1} e^x + C$ (b) $\log(1 + e^{2x}) + C$ (c) $\tan^{-1} e^{2x} + C$ (d) None of these

13. $\int \left(\frac{1}{x} - \frac{1}{x^2} \right) e^x \, dx =$ _____

(a) $\frac{1}{x^2} e^x + C$ (b) $\frac{1}{x} e^x + C$ (c) $-\frac{1}{x} e^x + C$ (d) none of these

14. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{3/2} - 2x\left(\frac{dy}{dx}\right)^{2/3} + 6y = 0$ is

(a) 2 & 9 (b) 2 & 3 (c) 2 & 6 (d) 2 & 1

15. If $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + 2\vec{j} + \vec{k}$, then $\cos\theta =$

(a) $\frac{6}{7}$ (b) $\frac{5}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{2}$

16. The value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$ is

(a) 0 (b) 1 (c) -1 (d) 3

17. The area of a triangle whose two sides are represented by the vectors $2\hat{\imath}$ *and* $-3j$ is

(a) 0 (b) 3 (c) 6 (d) 1

18. The projection of the vector $2\hat{\imath} - \hat{j} + \hat{k}$ on the vector $\hat{\imath} - 2\hat{j} + \hat{k}$ is

(a) $4/\sqrt{6}$ (b) $5/\sqrt{6}$ (c) $4/\sqrt{3}$ (d) $7/\sqrt{6}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A) : The function $y = [x(x-2)]^2$ attains its local maxima or minima at the points x = 1 & 2

Reason (R) : The slope of the tangent at the points of local maxima or minima is 0.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false.

(d) A is false but R is true.

20. Assertion (A) : The number of arbitrary constants in the solution of differential equation $\frac{d^2y}{dx^2} - 2x = 0$ are 2.

Reason (R) : The solution of a differential equation contains as many arbitrary constants as the order of differential equation.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Find $\frac{d^2y}{dx^2}$, if $y = \log x$

OR

Find $\frac{dy}{dx}$ if $y = \sin(\tan^{-1}e^x)$.

22. The radius of a circle increases at a rate of 3 cm/sec. What is the rate of increase of its area at the instant when radius of circle is 10 cm.

23. Evaluate $\int_1^4 (x^2 + 2x)dx$

OR

Evaluate: $\int \frac{\sec x\, dx}{\log(\sec x + \tan x)}$

24. Find $|\vec{x}|$, if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

25. Find the direction ratio and direction cosines of a line parallel to the line: $\frac{2-x}{2} = \frac{-y+3}{-7} = \frac{z+1}{2}$

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, find $\frac{dy}{dx}$.

OR

If $x\sqrt{1+y} + y\sqrt{1+x} = 0, for, -1 < x < 1$,.prove that: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

27. Find the interval in which the function $f(x) = \sin x - \cos x$, $0 \le x \le 2\pi$ is

(a) Increasing (b) decreasing

28. Evaluate: $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$

OR

Evaluate: $\int_{-5}^{5} |x + 2|\, dx$

29. Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration

30. If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.

31. Find the value of k for which the following lines are perpendicular to each other:
$\frac{x+1}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1}$; $\frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

33. Solve the differential equation $cos^2x\frac{dy}{dx} + y = tanx$; $y(0) = 1$.

OR

Show that (x-y) dy=(x+2y)dx is a homogenous differential equation. Also, find the particular solution of the given differential equation given that y = 0 when x = 0.

34. Evaluate: $\int_0^{\pi} \log(1 + \cos x)\, dx$

OR

Evaluate: $\int \frac{(3sinx-2)\,cosxdx}{13-cos^2x-7sinx}$ dx

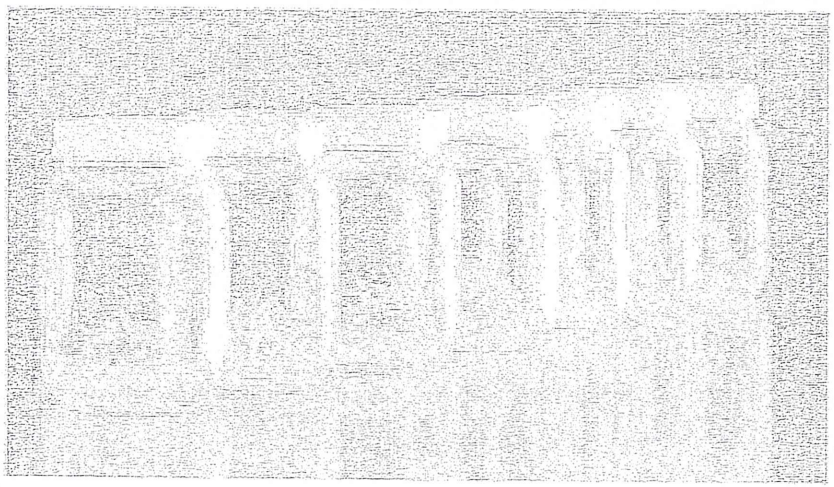
35. Find the coordinates of the foot of the perpendicular drawn from the point (5, 4, 2) to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Also find the equation and length of the perpendicular.

SECTION E

(This section comprises of 3 case based questions of 4 marks each with two or three sub-parts.)

36. Case-Study 1: Read the following passage and answer the questions given below.

The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number of bacteria present at that time. Given that the number of bacteria triples in 5 hours.



(i) If 'N' is the number of bacteria present at any time 't', write the differential equation corresponding to the above situation. [1Mark]

(ii) Find the the general solution of the above differential equation. [2 Marks]

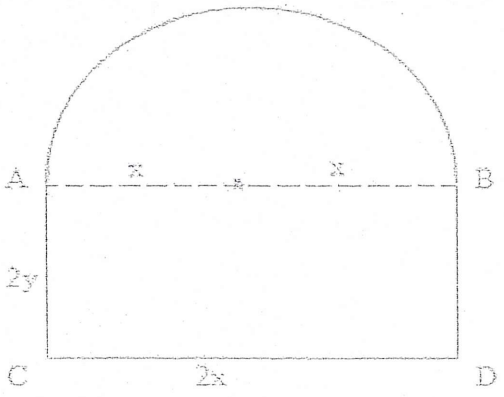
(iii) If the initial count of bacteria is 100000, find the count of bacteria after 10 hours.

OR

In how many hours will the bacteria count become 10 times. [1Mark]

37. Case-Study 2: Read the following passage and answer the questions given below.

Mr Mehta, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below :



(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then what is the relation between the variables x and y ? [1Mark]

(ii) What is the combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x. [1Mark]

(iii) What is the maximum value of area A of the whole window ? [2 Marks]

OR

The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.For this to happen, what should be the length of rectangular portion of the window?

38. Case-Study 3: Rohan had a doubt in a problem of continuity and differentiability during Half Yearly examination. He asked his doubt to his friends Ankit and Mayank. He asked," Check whether the function $f(x) = |x - 3|$, $x \in R$ is continuous and differentiable at x = 3 or not."

Ankit told that the given function is continuous at x=3 but not differentiable at x=3 while Mayank told that the function is continuous and differentiable at x=3.

Now Rohan was in a state of confusion whether Ankit is correct or Mayank. So to clear his doubt, he asked the question to his teacher over phone and his teacher cleared his doubt.

Based on the above information, answer the following questions:

(i) Check whether the function $f(x) = |x - 3|$, $x \in R$ is continuous at x = 3 or not. [2 Marks]

(ii) Check whether the function $f(x) = |x - 3|$, $x \in R$ is differentiable at x = 3 or not. [2 Marks]